Geometry

INTRODUCTION

In the elementary grades, experiences in geometry should provide for the development of the concepts of shape, size, symmetry, congruence, and similarity in both two-dimensional and three-dimensional space. Experiences should begin with familiar objects and should utilize a wide variety of concrete materials to develop appropriate vocabulary and to build understanding. In middle school instruction, more formal generalizations in geometric relationships can be stressed. The activities provided in this section are designed to provide problem-solving experiences in geometry that help students develop understanding of geometric properties and relationships.

Why Teach Geometry?

Young children have considerable experience with geometry before entering school. They spend a great deal of time exploring, playing, and building with shapes. In their play experiences, children encounter relationships among shapes naturally. They sort and resort objects, make discoveries about how different blocks fit together, and learn about shapes that roll, slide, or do neither.

These initial investigations should be nurtured and extended in children's school learning of mathematics. In their classroom experiences, children should have opportunities to explore shapes and the relationships among them. They benefit from problem-solving situations that lead them to investigate patterns and structures in shapes and to develop reasoning processes in spatial contexts. They need experiences that relate geometry to ideas in measurement, number, and patterns. Through these kinds of activities, students grasp how mathematics adds to their understanding of the world.

Although elementary mathematics textbooks include sections on geometry, it's not uncommon for teachers to treat these sections as optional and to skip them or to present them to the children as if they were less important. There are several explanations for this. Because the major emphasis in elementary mathematics has been on the teaching of arithmetic skills, some teachers think that geometry in math instruction is not as significant for elementary students. Also, many teachers remember geometry as a high school subject that dealt mainly with formal proofs and complicated terminology. However, geometry is a significant branch of mathematics, the one most visible in the physical world.

Developing spatial ability has applications in everyday life, a fact that any adult encounters when having to figure quantities for wallpaper, floor covering, paint, fabric, lawn needs, or a myriad of other home projects. Geometric concepts and relationships are also essential to many branches of industry, the building trades, interior design, architecture, as well as other work situations. Geometry should be included as an integral part of the mathematics program.

A SAMPLE ACTIVITY — PENTOMINOES

The emphasis in this geometry activity is on informal, concrete experience, not on the symbolism and formal definitions that are the focus in many textbooks. *Pentominoes* calls on a different kind of reasoning than is needed for numerical tasks. In the classroom, children who are not generally considered to be good math students often enjoy success in these kinds of spatial experiences.

There are two aspects to this activity. One is searching for possible arrangements of squares, a geometric visualization task. The other is deciding when all possible arrangements have been found, which requires the use of logical reasoning.

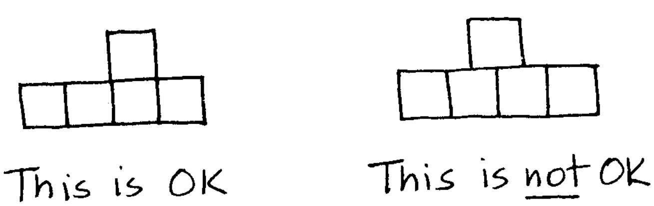
Materials

square tiles (about 1 inch on a side), five per student
paper ruled into squares the same size as the tiles, two sheets per group

Introducing

1. Present or review concepts.

Three points need to be presented here. First, demonstrate the rule for making shapes in which one whole side of each square touches at least one whole side of another. Draw the following examples on the board or overhead:



Second, explain how to decide if two shapes are the same or different. Cut the two shapes in question out of paper and move them about to see if one fits exactly on the other. You might have to flip one over, or rotate one, but if they can be made to fit, they are called *congruent* and are considered to be the same.

Finally, discuss the derivation of the word *pentomino*. Draw a picture of a domino on the board or overhead. It's made of two squares. A pentomino is a five-square version. A three-square version is a triomino; a tetromino is made from four squares.

2. Pose a part of the problem or a similar but smaller problem.

Ask: Suppose you were trying to find all the different arrangements of three squares. What shapes could you make? Have students try this. There are only two triominoes:  and . Have them try the same with four squares. (There are five possible tetrominoes.)

3. Present the problem to be solved.

Ask groups to investigate different ways to arrange five squares. Direct them to cut each of the pentominoes they find out of the graph paper provided.

1 - 1 x1+0
2 - 1 x2+0
3 - 2 x2+1
4 - 5 x2+2
5 - 12 x2+3
6 - 2?

4. Discuss the results.

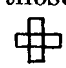
Ask for questions.

Exploring

Two situations typically arise during the exploration. Some groups cut out two shapes that are the same but believe they are different. In that case, comment that you notice that two of their shapes are the same, and leave them to find the congruent shapes.

The second situation arises when a group has found all they can. Usually they'll ask you if they have them all. Tell them that is for them to decide and encourage them to find some way to analyze their arrangements to see if they think their collection includes all that are possible. Also tell them this issue will be discussed after all the groups in the class have finished the activity.

Extension

Direct the group to sort their shapes into two sets, those that will fold into a box and those that will not. Demonstrate with the shape that looks like the Red Cross symbol:  Ask: Can you see how you could fold up the sides of this shape so that it would be a box without a lid? Which side do you think would be the bottom of the box, opposite the open side? Mark that with an X. Now try folding it to check your prediction. With each of the other shapes, predict whether or not you think it will fold into a box, marking with an X the side that will be the bottom.

Summarizing

Discuss their group processes. Ask students to report how they found different shapes and decided who would cut out the pieces.

Discuss how they knew when they had found all the pieces possible. This is a good opportunity to discuss the issue of answers. If you had told them in advance that there were 12 shapes, you would have limited their opportunity for problem solving. Students benefit from hearing you reinforce the importance of problem solving. The goal in this activity is not just to find all the shapes; it is also to know when you've found them.

In classrooms, children often learn to depend on the quicker thinkers to provide answers. They also learn to depend on the teacher for finding out if they are right or wrong. It's valu-

able for children to learn to become self-reliant in their thinking processes. You can support this by keeping the emphasis on the problem-solving process and by not providing answers.

Discuss the challenge if all groups have had a chance to explore it. If not, discuss it at a later time, after students have had sufficient time to tackle it.

Extensions

The Pentomino Game

Students each make a set of pieces from sturdy paper and a gameboard that is a 5-by-12 squared sheet, with squares matching the size of the squares they used for the pentominoes. As an individual puzzle, they try to fit all 12 pieces onto the board. As a two-person game, players take turns placing pieces on the board. The object is to be the last player to play a piece, making it impossible for the opponent to fit in another. In this second version, all the pieces do not have to be used.

The Factory Box Problem

Someone in a factory bought lots of cardboard that measured five squares by four squares. They figured that each sheet of cardboard could be cut into four pieces so each piece would fold into a topless box. How could the sheet be cut?

Milk Carton Geometry

Save school milk cartons. (Rinse them well!) When you've got enough so each student can have several, let them cut the tops off so all are topless boxes. Then students try to cut them so they lie flat in the different pentomino shapes.

Pentomino One-Difference Loop

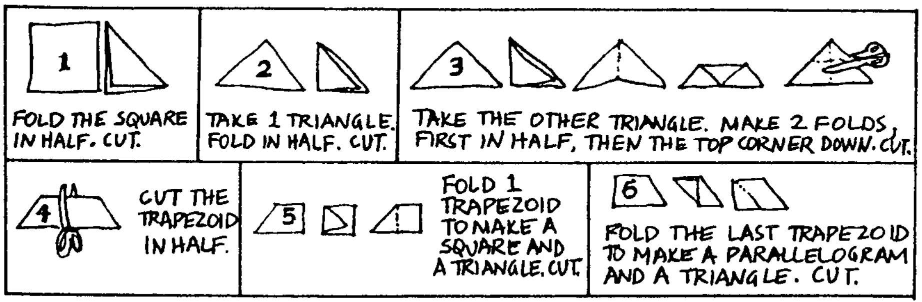
Arrange the pentomino pieces into a loop, so that only one square needs to be moved to change a shape into the one next to it.

ACTIVITIES

WHOLE CLASS: The Tangram Puzzle

You need: 6-inch squares of construction paper, one per student
scissors, one pair per student

The Tangram is cut from a square. Having children each cut their own is a good lesson in following directions. Also, children are then convinced that the pieces truly go back together to make a square. The directions below show how to cut the square into the seven pieces: two pairs of congruent triangles, one middle-sized triangle, one square, and one parallelogram.



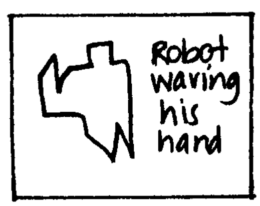
Using just the three smallest triangles, make a square. Then use those same pieces to make a triangle, a rectangle, a trapezoid, and a parallelogram. Then use the five smaller pieces (all but the two large triangles) to make the same shapes. Repeat with all seven pieces. Record on a chart as shown.

3 small triangles					
5 small pieces					
all 7 pieces					

Extensions:

1. *Area and Perimeter.* Compare the areas of the square, the parallelogram, and the middle-sized triangle. Then compare their perimeters.

2. *Tangram Puzzle Cards.* Have children explore making shapes using all seven pieces. When they find ones that please them, have them draw around the outline of the shape on drawing paper. They then name it, sign it, and put it in a class Tangram box so others can try to fit their pieces into that shape. Students sign their names on the backs of each other's puzzles as they solve them.



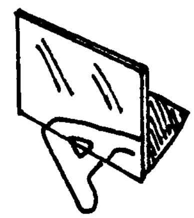
3. *Using All Seven.* There are 13 different convex shapes you can make with the seven pieces of the Tangram. Find them.

4. *Making Squares.* You can show a square with just one piece of the Tangram or by using all seven. What about using two, three, four, five, or six? One of those is impossible. Which is it, and why isn't it possible?

That's Just Half the Story

You need: a small rectangular mirror

Some letters have mirror symmetry. That means when you place the mirror on them, you can see the whole letter — half on your paper and half in the mirror. Which letters work this way? Which work more than one way? Record the uppercase letters on a chart. Put in dotted lines to show where you placed the mirror.



These work one way.	These work more than one way.	These do not have mirror symmetry.
A	X	F

Interior Regions

Which uppercase letters have interior regions? That means if you built a fence in the shape of that letter, it would keep your dog inside. Record on a chart as shown.

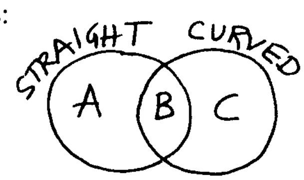
YES	NO
P	E

Straight or Curved?

Which uppercase letters have only straight line segments? Which have only curves? Which have both? Record like this:

STRAIGHT	CURVED	BOTH
A	C	B

Then put your results on a Venn diagram, like this:

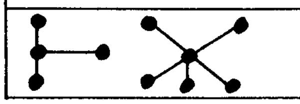
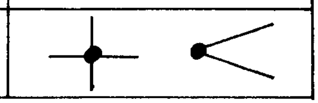


No-Lift Letters

It's possible to draw this shape  without lifting your pencil and without retracing any line. Try it. Then investigate the alphabet. Which uppercase letters can you write without lifting your pencil and without retracing any line? Record.

More No-Lift Letters

You can tell if a letter will work by looking at the points where line segments meet or end (the vertices) and by seeing how many of these points have an odd number of line segments meeting there (odd vertices).

ODD VERTICES	EVEN VERTICES
	

Count the odd vertices for all the letters than can be written without lifting your pencil or retracing. What's the pattern?

LETTER	B	C	D		
ODD VERTICES	2	2	0		

Extensions:
1. Try all the activities with lowercase letters.
2. Some letters have rotational symmetry. That means they are the same when turned upside down. Make a list of these.

Geometry Building

You need: a partner or small group
an identical set of materials for each person — blocks, toothpicks, squares, etc.
walls (from books or binders) so each person has a working space that no one else can see

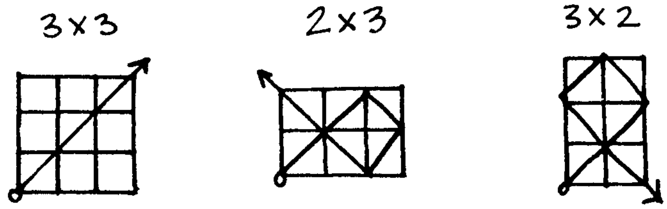
One person builds a structure using some or all of the materials and then describes it so the others can build it. The others can ask questions at any time. Finally, lift your walls to see if the structures you built are the same. Try it again so all students have a chance to be the describer. Discuss the language used, focusing on what was useful and what was not.

Pool Hall Math

(Adapted from *Mathematics: A Human Endeavor* by Harold Jacobs, Freeman Publishing Company)

You need: centimeter squared paper (see blackline masters)

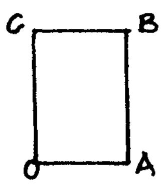
A pool table is a rectangle or square of any dimensions. The ball always starts at the lower left corner. It moves by going to the opposite corner of each square it enters and keeps moving until it reaches a corner. Examples:



If the ball goes through *every* square on the table, then that pool table is **INTERESTING.** If the ball doesn't go through every square, the table is **BORING.** Find 10 interesting and 10 boring pool tables. Record. Can you find a pattern for predicting whether a pool table is interesting or boring before you test it?

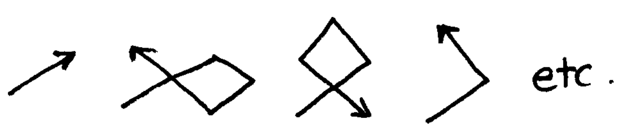
Extensions:

1. *Exit Corners.* Examine where the ball leaves each pool table. Some go out at A; others at B or C. Find 10 different pool tables for each exit corner. Can you find a way to predict the exit corner?



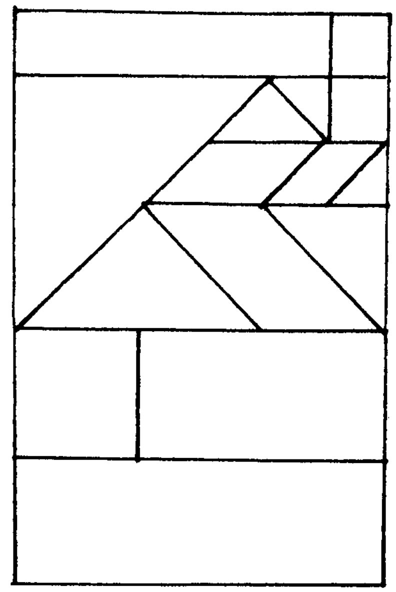
A	B	C
3x2	3x3	2x3

2. *Paths.* The ball takes different paths on different tables. Find all the different patterns for the paths the balls take.



Area and Perimeter

You need: a partner or small group
a sheet of shapes as shown, duplicated on tag (see blackline masters)
a large piece of chart paper
scissors

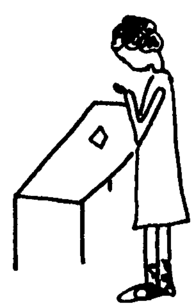


Cut out the shapes on the Area and Perimeter sheet. Use these pieces to do the following:

1. Order the pieces according to their areas.
2. Then compare the perimeters of the pieces.
3. Display results to show the relationships between the areas and perimeters.
4. Write statements that describe what you discovered.

Extensions:

1. *Sorting by Shape.* Compare the areas and perimeters of each of the triangles, and then of the squares, and then of the rectangles and parallelograms. Write statements that describe what you discovered.
2. *Similar Shapes.* Similar shapes have the same shape but are different sizes. You can informally test to see if two shapes are similar by "sighting" with one. Here's how. Place the large square on the table. Stand up so you're looking down on it. Hold the small square in one hand, close one eye, and move the square up and down until it exactly covers the larger square. Because it can cover it exactly, they are similar. Will all squares be similar? Test the triangles, parallelograms, and rectangles the same way. Which are similar and which are not?



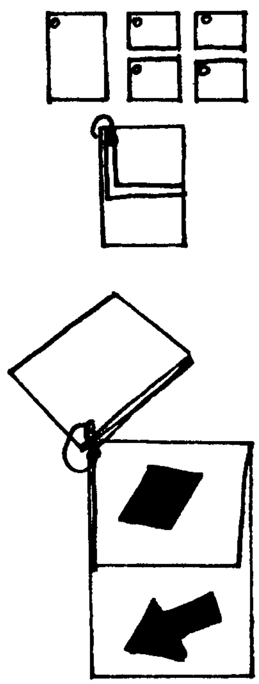
3. *Arrangements.* All of the pieces on the sheet fit perfectly into a rectangle. Could all of them be arranged into a triangle? A square? A parallelogram? Find ways to arrange them into other shapes.

Mirror Cards

You need: three 3-by-5-inch index cards
a small piece of yarn or string
markers
mirrors
a hole punch

Follow the directions to make mirror cards:

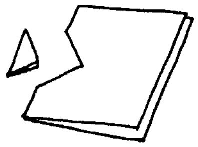
1. Cut two of the index cards in half.
2. Punch holes in the same corner of the four half-cards and in the card that wasn't cut. Put the stack of cut cards on top and tie all five cards together with a piece of yarn. Write your name on the back of each card.
3. On the bottom half of the uncut card, make a design.
4. Put a mirror anywhere on the design, and draw on the half-card what you see both on your design and in the mirror.
5. Flip that half up. Move the mirror and draw what you see now on the second half-card.
6. Continue until you've drawn five designs in addition to your original one.
7. Try to solve others' cards, using the mirror to figure out where the creator placed it to get the design drawn. Write your name on the back of the uncut card to show you've solved the set. If you have trouble solving one, talk with the person who drew it.



The Fold-and-Cut Investigation

You need: scissors

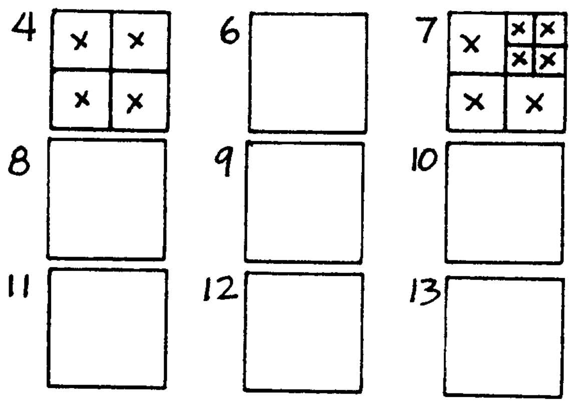
Fold a piece of paper in half. Cut out a small shape on the fold. Try to draw what the paper will look like when you unfold it. Then unfold and compare. Do this at least five times. **Note:** Try deciding on a shape first and then cutting to see if you get it.



Try the same activity, but fold the paper twice before cutting.

Square Partitioning

Draw nine squares on a sheet of paper. Number them as shown. Then divide each square into the number of smaller square regions written beside it.
Examples:



Extensions:
1. *There's More Than One Way to Cut a Square.* Find all the different ways you can partition a square into a given number of smaller squares.
2. *What's Wrong with 5?* Notice that 5 was omitted. How come?
3. *Going Further.* Continue the activity for larger numbers.

The Banquet Table Problem

You need: Color Tiles
squared paper, centimeter or half-inch (see blackline masters)

A banquet hall has a huge collection of small square tables that fit together to make larger rectangular tables. Arrange tiles to find the different numbers of people that can be seated if 12 small tables are used. Do the same if 24 are used. Record on squared paper.

Extensions:
1. *The 100 Table Problem.* If 100 small square tables are arranged into a large rectangular table, find the most and least numbers of people that can be seated.
2. *Banquet Cost.* If the banquet hall charges by the number of square tables used, what's the least expensive way to seat 16 people? 50 people? 60? 100? Any number?

Introductory Explorations with Pattern Blocks

You need: Pattern Blocks

Try the following introductory explorations to become familiar with Pattern Blocks.

1. Make a floor, covering as large an area as you'd like. Try this using blocks that are different kinds. Then see if you can do it using only one kind of block. Will all blocks work?
2. Make a straight road using only one kind of block. Can you do this with each different block? Which of your roads can you make turn a corner?
3. Make a design with the Pattern Blocks on a piece of heavy paper. Trace around the outside of your design. List how many of each block you used. Exchange papers with classmates to see if you can fit the proper pieces into each others' designs.
4. Try building a larger triangle using only green triangles. Try building a larger square using only orange squares. Try the same with each of the other pieces. Which work and which do not?
5. Try building a shape exactly the same as (congruent to) the yellow hexagon using only green triangles. Try this with each of the other pieces. Which work and which do not?
6. If the area of the green triangle has the value of one unit, find the value of the area of the blue, red, and yellow pieces. Do the same with the area of the blue diamond as one unit, and then again with the red and yellow pieces as one unit.
7. Compare the areas of the orange and white pieces. Convince a friend of your comparison.

Hexagon Fill-In Puzzle

You need: Pattern Blocks
large hexagon shape drawn with each side double the length of the yellow hexagon (see blackline masters)

You need 6 blocks to fill the hexagon shape with as few pieces as possible — 3 yellow hexagons and 3 blue parallelograms. (Try it.) To fill it with as many blocks as possible, you would use 24 green triangles. Explore the following:

1. Can you find ways to fill the hexagon shape with each number of blocks from 6 to 24 (7, 8, 9, 10, etc.)? Record.
2. For each of the numbers possible, find different ways to fill the shape. Record. (A different way means a different collection of blocks, not a different arrangement of the same blocks.)

Extension: In how many different ways can the hexagon shape be filled using Pattern Blocks?

Hexiamonds

You need: green triangles from the Pattern Blocks
Pattern Block triangle paper (see blackline masters)

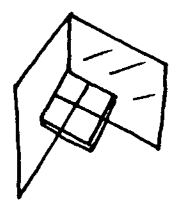
Hexiamonds are shapes made from six equilateral triangles arranged so that each triangle touches at least one other. Whole sides must touch. Use the green triangles to find all the different (noncongruent) hexiamonds. Cut them out of the triangle paper to verify that they are different. Record your solutions. Explain how you know you have found them all.

Angles with Pattern Blocks and Hinged Mirrors

You need: Pattern Blocks
hinged mirrors
5-by-8-inch index card with a dot and line on it, as shown below

The size of an angle is a measure of rotation, and degrees are used to measure angles.

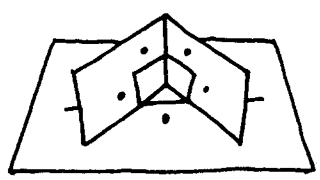
Part 1. Figure out how many degrees there are in the angles formed by the corners of each Pattern Block. Use the following procedure:



1. Place a corner of a block in the hinged mirrors.
2. Close the mirrors so the corner nestles snugly.
3. Use Pattern Blocks to build the design.
4. Sketch or trace it.
5. Figure the degrees in the nestled corner by dividing the number of blocks in the design into 360 degrees. Label your drawing to show the number of degrees in each angle.

Part 2. Construct other-size angles using the Pattern Blocks, hinged mirrors, and index card. Experiment with the following procedures:

1. Use combinations of Pattern Blocks. For example, the orange square and blue parallelogram can be put together to make an angle that is 90 degrees plus 60 degrees.
2. Use the hinged mirrors and the index card. For example, place the hinged mirrors so you see five dots and a pentagon. Trace along the base of the hinged mirrors to draw the angle created. Divide 360 degrees by 5 to figure the size of the angle traced. Label the angle.



3. Combine to make angles from blocks and mirrors.

Extension: What size angles cannot be constructed using Pattern Blocks and hinged mirrors?

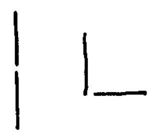
Explorations with Four Toothpicks

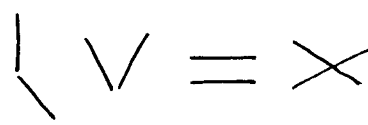
You need: a partner or small group
flat toothpicks, about 80
toothpick dot paper (see blackline masters)

Investigate all the different ways to arrange four toothpicks by following two rules:

1. Each toothpick must touch the end of at least one other toothpick.
2. Toothpicks must be placed either end to end or to make square corners.

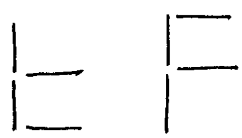
These are OK. These are <u>not</u> OK.





If a shape can be rotated or flipped to look like another shape, both shapes are the same. It's helpful to draw the shape in question on a piece of paper, flip it, and hold it up to a window to see if it's the same as another shape.

These are the same.



Record your shapes on dot paper. When you find all 16 shapes, cut your dot paper to make a set of cards.

Play a game. You need your cards and four toothpicks. Choose a starting card and make the pattern with the toothpicks. Deal the remaining cards to each player. (Discard extras so each player has the same number of cards.) Players place their cards face up so all are visible. In turn, each plays a card that shows a pattern that can be made from changing the position of exactly one toothpick on the pattern shown. Players help each other with moves and discuss patterns. Players pass if they can't play. The player who uses all his or her cards first is the winner for that round.

Extension: *The Put-in-Order Problem.* Arrange the cards so that each pattern can be made from the previous pattern by changing the position of just one toothpick. Can the cards be arranged in a continuous loop? Can they be arranged in more than one way?

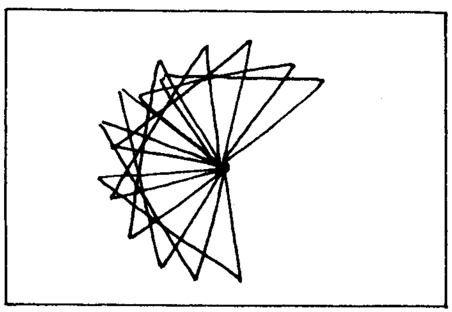
WHOLE CLASS: The Four-Triangle Problem

You need: 3-inch squares of construction paper in two colors
scissors
tape or paste
newsprint

In this investigation, children explore shapes made from construction paper triangles. Begin by showing children how to cut a square in half on the diagonal to make two triangles. Have them explore the different ways to put the triangles together, following the rule that two edges the same size must be matched. Discuss the possibilities with the class. (There are three — square, triangle, and parallelogram.) Then continue with the following explorations:

1. Have children, in pairs, investigate the different shapes they can make using four triangles. They should tape the triangles together or paste them on newsprint. Have many squares available for their exploration.
2. On a class chart called *Polygons,* organize the shapes the children found. To introduce the standard vocabulary, label columns: *triangle, quadrilateral, pentagon,* and *hexagon*. Have children post their shapes.
3. Have children choose one four-triangle shape. Using two triangles of each color, they make the shape with different arrangements of the colors.

Extension: *Rotating Shapes.* Children choose a four-triangle shape, trace it on tagboard, and cut it out. They draw a dot near the middle of a 12-by-18-inch sheet of drawing paper. They place a corner of their shape on the dot and trace. Then they rotate the shape and trace again. They continue rotating and tracing.



Penticubes

You need: *interlocking cubes (Snap or Multilink cubes)*
squared paper, the same size as faces of the cubes (see blackline masters)
isometric dot paper (see blackline masters)

This activity extends the pentomino exploration into three dimensions. Penticubes are three-dimensional shapes made from five cubes each. See how many you can construct. The following are ideas for follow-up explorations.

1. *Penticube Jackets.* Using paper ruled into squares the same size as the faces of the cubes, cut a jacket to fit a penticube. (Jackets are nets that fold to cover exactly all faces of the penticube.)
2. *More Penticube Jackets.* Take one penticube and find different possible jackets. (Is the number of possible jackets the same for each penticube?)
3. *Surface Area.* Compare the surface area of different penticubes. What do you notice about the shapes of penticubes with different surface areas?
4. *Penticube Riddles.* On squared paper, draw three views of a penticube — top, bottom, and side. Staple the drawings to a paper bag that holds the actual penticube. Others try to build the shape from the drawings and check their construction with the structure in the bag.
5. *Perspective Drawing.* Use isometric dot paper to draw penticubes.
6. *Building Rectangular Solids.* Put together several of the same or different penticubes to make rectangular solids. Investigate which dimensions of rectangular solids are possible to build.

WHOLE CLASS: Explorations Using the Geoboard

You need: geoboards, one per student
rubber bands

The following ideas are useful for beginning investigations.

1. Have children make numerals on the geoboard.
2. Have children make their initials. Ask them to make other letters.
3. Ask children to make something that can fly. Have students show what they made. Discuss. For example: Joe made a rocket. Did anyone else make a rocket? How are the rockets alike? How are they different?
4. Ask children to use just one rubber band and to make a shape that touches four pegs with one peg inside. It helps to describe this as making a fence that has four fenceposts and one tree inside the fence. Have children check with their neighbors to see that everyone has done it correctly and if there are different solutions. Continue: Make a shape that touches five pegs with zero inside; make a fence with five fenceposts and two trees inside, etc.
5. Ask children to use one rubber band and to make a shape that is not a square and looks the same on whichever side the geoboard rests. Ask: How many different shapes did we make in the class?

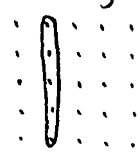
Explorations Using the Geoboard

You need: a partner or small group
geoboards
rubber bands

Each student finds a solution for the first direction. In your group, check each other's solutions, compare results, and discuss similarities and differences. Then continue with the next direction.

1. Make a shape that touches five pegs. (Think of the rubber band as a fence, and the pegs it touches as fenceposts.) Then try shapes that touch six and four pegs.
2. Make a shape that has three pegs inside. (That means if the shape is a fence the pegs inside are trees inside the fence.)
3. Make a shape that has ten pegs outside it, not touching the rubber band. (Think of them as trees growing outside the fence.)
4. Make a shape that has five fenceposts with three trees inside. Then try six fenceposts with two trees inside and three fenceposts with two trees inside. A challenge: Are there any combinations of fenceposts and trees that are not possible?
5. Use two rubber bands. Use each to make a line segment so the two line segments touch a total of nine pegs.
6. Repeat item 5 again, this time finding different ways to make the line segments (a) parallel, (b) intersecting, (c) perpendicular, (d) the same length.

a line segment

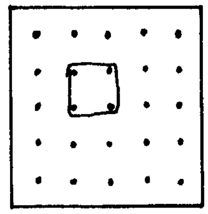


7. Make a triangle with one square corner and no two sides the same length.
8. Make a four-sided polygon with no parallel sides.
9. Make a four-sided polygon with all sides different lengths.
10. Make a four-sided polygon with no square corners but with opposite sides parallel. (What is this polygon called?)
11. Make a four-sided polygon that is not a square, not a rectangle, not a parallelogram, and not a trapezoid.
12. Make two shapes that have the same shape but are different sizes and are not squares.

Pick's Theorem

There is a function called Pick's Theorem (named after the mathematician who discovered it) that enables you to find the area of any shape on the geoboard from the number of pegs on the perimeter of the shape (P) and the number of pegs inside the shape (I). The following sequence of activities suggests a way to find this theorem.

1. The square with an area of 1 has four pegs on its perimeter and zero pegs inside. Investigate other shapes with four pegs on the perimeter and zero pegs inside. Compare their areas. What about shapes with four pegs on the perimeter and one peg inside? Two pegs inside? Three? Four? Five? Etc.? Write a formula that describes the relationship.



Remember: P = pegs on the perimeter
I = pegs inside
A = area of the shape

P	I	A
4	0	1
4	1	
4	2	
4	3	
4	4	
⋮	⋮	

2. Do the same investigation for shapes with other numbers of pegs on the perimeter.

P	I	A
3	0	
3	1	
3	2	
3	3	
⋮	⋮	

P	I	A
5	0	
5	1	
5	2	
5	3	
⋮	⋮	

P	I	A
6	0	
6	1	
6	2	
6	3	
⋮	⋮	

3. Now investigate patterns for the areas of shapes when the number of inside pegs stays constant and the number of pegs on the perimeter varies. For example:

P	I	A
3	0	
4	0	
5	0	
6	0	
7	0	
⋮	⋮	

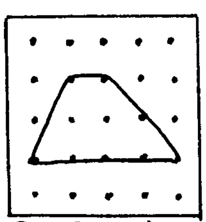
P	I	A
3	1	
4	1	
5	1	
6	1	
7	1	
⋮	⋮	

P	I	A
3	2	
4	2	
5	2	
6	2	
7	2	
⋮	⋮	

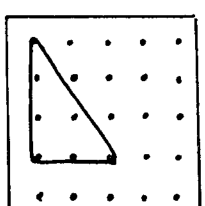
etc.

4. Finally, can you find a master formula that allows you to figure the area (A) for any combination of pegs on the perimeter (P) and pegs inside (I)? That's Pick's Theorem.

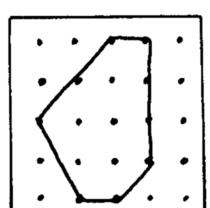
Just to check your formula:



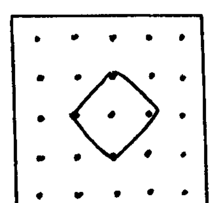
P=8 I=2 A=?



P=6 I=1 A=?



P=9 I=5 A=?



P=4 I=1 A=?

Area on the Geoboard

You need: a geoboard
rubber bands

Find the area of each shape.